WEEKLY TEST TYJ MATHEMATICS SOLUTION 15 SEPT 2019

31. (b) We have $\sqrt{2}+\sqrt{8}+\sqrt{18}+\sqrt{32}+......$

$=1\sqrt{2}+2\sqrt{2}+3\sqrt{2}+4\sqrt{2}+........$

$=\ \sqrt{2}[1+2+3+4+..........\text{upto 24 terms}]$

32. (d) Given that $S_n = nA + n^2B$

Putting $n = 1, 2, 3, ...,$ we get

$S_1 = A+B, S_2 = 2A+4B,\ S_3 = 3A+9B$

....

....

Therefore $T_1 = S_1 = A+B,\ T_2 = S_2 - S_1 = A+3B,$

$T_3 = S_3 - S_2 = A+5B$,

....

....

Hence the sequence is $(A+B),(A+3B),(A+5B),........$

Here $a = A+B$ and common difference $d = 2B$.

33. (d) $\log_3 2, \log_3(2^x-5)$ and $\log_3\left(2^x-\frac{7}{2}\right)$ are in A.P.

$\Rightarrow\ 2\log_3(2^x-5)=\log_3\left[(2)\left(2^x-\frac{7}{2}\right)\right]$

$\Rightarrow\ (2^x-5)^2 = 2^{x+1}-7\ \Rightarrow\ 2^{2x}-12\,.\,2^x-32=0$

$\Rightarrow\ x = 2, 3$

But $x=2$ does not hold, hence $x=3$.

34. (a) $p\{a+(p-1)d\}=q\{a+(q-1)d\}$

$\Rightarrow\ a(p-q)+(p^2-q^2)d+(q-p)d=0$

$\Rightarrow\ (p-q)\{a+(p+q-1)d\}=0$

$\Rightarrow\ a+(p+q-1)d=0\ \Rightarrow\ T_{p+q}=0$, $\{\because p\neq q\}$.

35. (c) $T_m = a+(m-1)d=\frac{1}{n}$

and $T_n = a+(n-1)d=\frac{1}{m}$

On solving $a=\frac{1}{mn}$ and $d=\frac{1}{mn}$

$\therefore\ T_{mn}=a+(mn-1)d=\frac{1}{mn}+(mn-1)\frac{1}{mn}=1$

36. (c) Let S_n and S'_n be the sums of n terms of two A.P.'s and T_{11} and T'_{11} be the respective 11^{th} terms, then

$$\frac{S_n}{S'_n}=\frac{\frac{n}{2}[2a+(n-1)d]}{\frac{n}{2}[2a'+(n-1)d']}=\frac{7n+1}{4n+27}$$

$$\Rightarrow\ \frac{a+\frac{(n-1)}{2}d}{a'+\frac{(n-1)}{2}d'}=\frac{7n+1}{4n+27}$$

Now put $n=21$,

we get $\frac{a+10d}{a'+10d'} = \frac{T_{11}}{T'_{11}} = \frac{148}{111} = \frac{4}{3}$.

Note : If ratio of sum of n terms of two A.P.'s are given in terms of n and ratio of their p^{th} terms are to be found then put $n = 2p - 1$. Here we put $n = 11 \times 2 - 1 = 21$.

37. (c) Let $S_{Even} = 2+4+6+8+........\infty$ .....(i)

and $S_{Odd} = 1+3+5+7+9+.......\infty$ .....(ii)

Sum $S_E = \frac{n}{2}[4+(n-1)2] = \frac{n}{2}[2n+2] = \frac{n}{2}2(n+1)$

and $S_O = \frac{n}{2}[2+(n-1)2] = \frac{n}{2}(2n)$

Now $\frac{S_E}{S_O} = \frac{(n+1)}{n}$ or $S_E : S_O = (n+1) : n$.

38. (a) n^{th} term of the series is $20+(n-1)\left(-\frac{2}{3}\right)$.

For sum to be maximum, n^{th} term ≥ 0

$\Rightarrow 20+(n-1)\left(-\frac{2}{3}\right) \geq 0 \Rightarrow n \leq 31$

Thus the sum of 31 terms is maximum and is equal to

$\frac{31}{2}\left[40+30\times\left(-\frac{2}{3}\right)\right] = 310$.

39. (a) We have $(x+1)+(x+4)+.........+(x+28) = 155$

Let n be the number of terms in the A.P. on L.H.S. Then $x+28 = (x+1)+(n-1)3 \Rightarrow n = 10$

$\therefore (x+1)+(x+4)+......+(x+28) = 155$

$\Rightarrow \frac{10}{2}[(x+1)+(x+28)] = 155 \Rightarrow x = 1$.

40. (b) $T_5 = ar^4 = \frac{1}{3}$ .....(i)

and $T_9 = ar^8 = \frac{16}{243}$ .....(ii)

Solving (i) and (ii), we get $r = \frac{2}{3}$ and $a = \frac{27}{16}$

Now 4^{th} term $= ar^3 = \frac{3^3}{2^4}.\frac{2^3}{3^3} = \frac{1}{2}$.

41. (a) Series is a G.P. with $a = 0.9 = \frac{9}{10}$ and $r = \frac{1}{10} = 0.1$

$\therefore S_{100} = a\left(\frac{1-r^{100}}{1-r}\right) = \frac{9}{10}\left(\frac{1-\frac{1}{10^{100}}}{1-\frac{1}{10}}\right) = 1 - \frac{1}{10^{100}}$.

42. (b) Given series $6+66+666+........+$ upto n terms

$= \frac{6}{9}(9+99+999+.....$ upto n terms)

$= \frac{2}{3}(10+10^2+10^3+..........+$ upto n terms $-n$)

$= \frac{2}{3}\left(\frac{10(10^n-1)}{10-1} - n\right) = \frac{1}{27}[20(10^n-1)-18n]$

$= \frac{2(10^{n+1}-9n-10)}{27}$.

43. (b) Let G_1, G_2, G_3, G_4, G_5 be the G.M.'s are inserted between 486 and 2/3. So total terms are 7.

$T_n = ar^{n-1} \Rightarrow 2/3 = 486\,(r)^6 \Rightarrow r = 1/3$

Hence 4^{th} G.M. will be, $T_5 = ar^4 = 486(1/3)^4 = 6$.

44. (a) Since the series are G.P., therefore

$$x = \frac{1}{1-a} \Rightarrow a = \frac{x-1}{x} \text{ and } y = \frac{1}{1-b} \Rightarrow b = \frac{y-1}{y}$$

$$\therefore 1 + ab + a^2b^2 + \text{..........}\infty = \frac{1}{1-ab}$$

$$= \frac{1}{1 - \frac{x-1}{x} \cdot \frac{y-1}{y}} = \frac{xy}{x+y-1}.$$

45. (a) $4^{1/3}.4^{1/9}.4^{1/27}......\infty$

$$\therefore S = 4^{1/3+1/9+1/27}......\infty$$

$$\Rightarrow S = 4^{\left(\frac{1/3}{1-1/3}\right)} = 4^{\frac{1/3}{2/3}} \Rightarrow S = 4^{1/2} \Rightarrow S = 2.$$